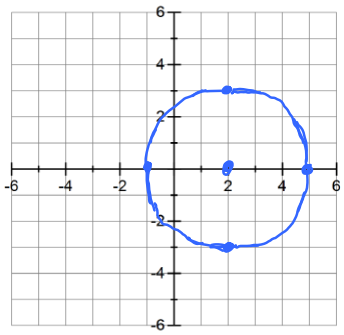
Exam 4 Review Key

Note Title 12/6/2012

1. $(x-2)^2 + y^2 = 9$

Center: $(2,0)$

$r = 3$



2. $r = 4$ center $(2,-5)$

$\boxed{(x-2)^2 + (y+5)^2 = 16}$

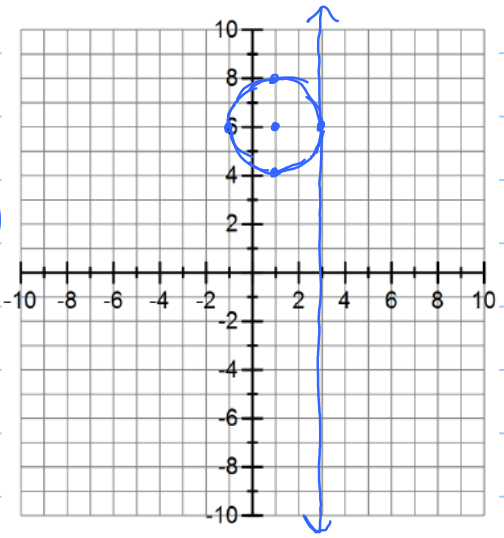
3. center: $(1,6)$

Tangent to $x=3$

(circle touches the line at one point)

$r = 2$

$\boxed{(x-1)^2 + (y-6)^2 = 4}$



4. $(2,-5)$ & $(6,1)$ are endpoints of a diameter

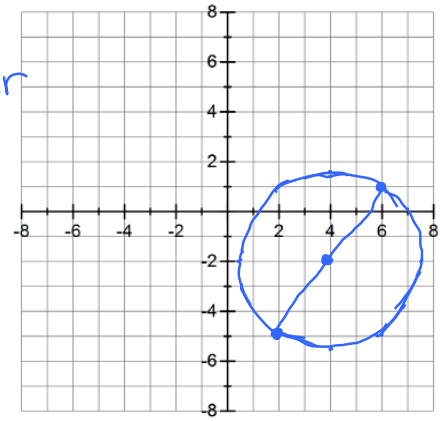
Center: $\left(\frac{2+6}{2}, \frac{-5+1}{2}\right)$

$(4,-2)$

radius = distance from center to endpoint.

$r = \sqrt{(6-4)^2 + (1-(-2))^2} = \sqrt{2^2+3^2} = \sqrt{13}$

$\boxed{(x-4)^2 + (y+2)^2 = 13}$



5. $x^2 + y^2 - 10x + 6y - 2 = 0$

$\frac{-10}{2} = -5 \quad (-5)^2 = 25 \qquad \frac{6}{2} = 3 \quad 3^2 = 9$

$(x^2 - 10x \quad) + (y^2 + 6y \quad) = 2$

$(x^2 - 10x + 25) + (y^2 + 6y + 9) = 2 + 25 + 9$

$(x-5)^2 + (y+3)^2 = 36$

center: $(5, -3)$
radius: 6

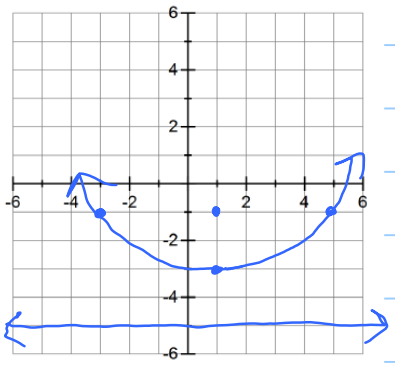
6. focus: $(1, -1)$
directrix: $y = -5$

vertex: $(1, -3)$

$a = 2$

$(x-h)^2 = 4a(y-k)$

$(x-1)^2 = 8(y+3)$



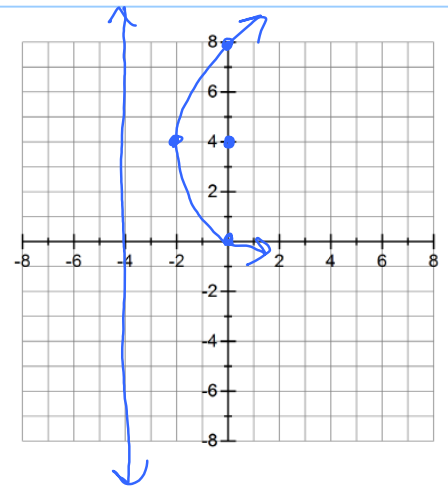
7. $(y-4)^2 = 8(x+2)$

vertex: $(-2, 4)$

$4a = 8 \Rightarrow a = 2$

focus: $(0, 4)$

directrix: $x = -4$



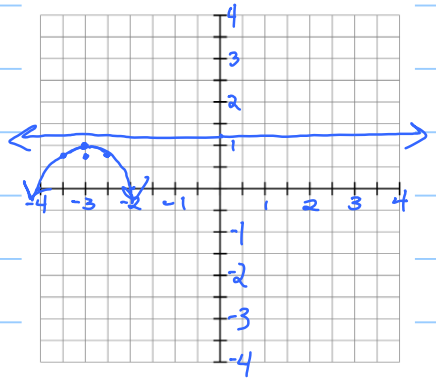
8. $(x+3)^2 = -(y-1)$

vertex: $(-3, 1)$

$-4a = -1 \quad a = \frac{1}{4}$

focus: $(-3, 3/4)$

directrix: $y = 1\frac{1}{4}$



9. $\frac{9x^2+4y^2}{36}=\frac{36}{36}$

$\frac{x^2}{4}+\frac{y^2}{9}=1$

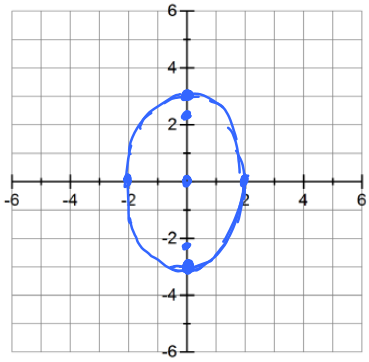
$\leftrightarrow 2 \quad \updownarrow 3$

$c^2=a^2-b^2$

$c^2=9-4=5$

$c=\sqrt{5}$

center: $(0,0)$
vertices: $(0,3)\ (0,-3)$
foci: $(0,\sqrt{5})\ (0,-\sqrt{5})$



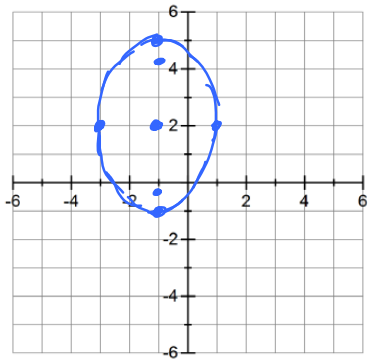
10. $\frac{(x+1)^2}{4}+\frac{(y-2)^2}{9}=1$

$\leftrightarrow 1 \quad \updownarrow 3$

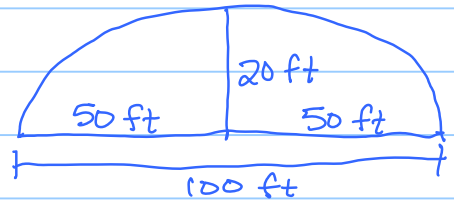
center: $(-1,2)$
vertices: $(-1,5)\ (-1,-1)$
foci: $(-1,2-\sqrt{5})\ (-1,2+\sqrt{5})$

$c^2=9-4=5$

$c=\sqrt{5}$



11.



$a=50 \quad b=20$

$c^2=a^2-b^2$

$c^2=2500-400=2100$

$c=\sqrt{2100}=10\sqrt{21}\approx 45.8\text{ ft}$

12. $\frac{16y^2-9x^2}{144}=\frac{144}{144}$

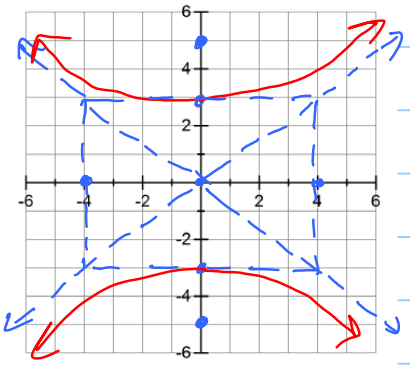
$\frac{y^2}{9}-\frac{x^2}{16}=1$

$\updownarrow 3 \quad \leftrightarrow 4$

$c^2=a^2+b^2=9+16=25$

$c=5$

center: $(0,0)$
vertices: $(0,3)\ (0,-3)$
foci: $(0,5)\ (0,-5)$



13. $\frac{(y+2)^2}{4} - \frac{(x-3)^2}{9} = 1$

$\updownarrow 2 \qquad \leftrightarrow 3$

center: $(3,-2)$

vertices: $(3,0)\ (3,-4)$

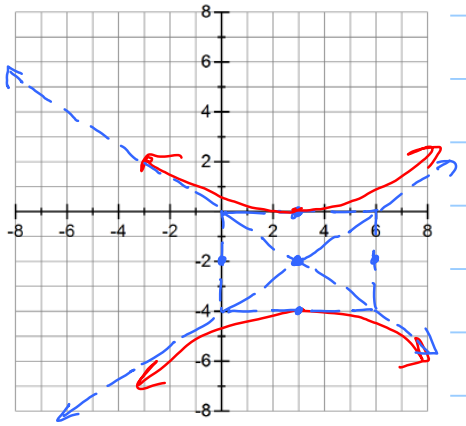
$c^2 = a^2+b^2 = 4+9 = 13$

$c = \sqrt{13}$

foci: $(3,-2-\sqrt{13})\ (3,-2+\sqrt{13})$

transverse axis: $x=3$

asymptotes: $y+2 = \pm\frac{2}{3}(x-3)$



14. vertices: $(0,\pm 5)$ $\qquad a=5 \qquad a^2+b^2=c^2$

foci: $(0,\pm 7)$ $\qquad c=7 \qquad 25+b^2=49$

center: $(0,0)$ $\qquad\qquad b^2=24$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$\boxed{\frac{y^2}{25} - \frac{x^2}{24} = 1}$

15. center: $(2,3)$

focus: $(0,3)$

vertex: $(1,3)$

$a=1 \quad c=2$

$a^2+b^2=c^2$

$1+b^2=4$

$b^2=3$

F V C V F

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\boxed{(x-2)^2 - \frac{(y-3)^2}{3} = 1}$

16. $\begin{cases} 4x-3y+z=-2 \\ 5y-z=6 \\ 2x+8z=-9 \end{cases}$

$$\left[\begin{array}{ccc|c} 4 & -3 & 1 & -2 \\ 0 & 5 & -1 & 6 \\ 2 & 0 & 8 & -9 \end{array}\right]$$

17. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $\begin{cases} x=-2 \\ y=4 \\ z=1 \end{cases}$

consistent
one solution: $(-2,4,1)$

18. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right]$ $\begin{cases} x=-2 \\ y=4 \\ 0=1 \end{cases}$

inconsistent
no solution $\emptyset$

19. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\begin{cases} x=-2 \\ y+3z=4 \\ 0=0 \end{cases}$ $y=-3z+4$

consistent, infinitely many solutions
$\{(x,y,z) \mid x=-2,\ y=-3z+4,\ z \text{ is any real \#}\}$

20. $\left[\begin{array}{ccc|c} 2 & 4 & 5 & -2 \\ 1 & 2 & 3 & 4 \\ 3 & 3 & 7 & 1 \end{array}\right]$ $R_2 = -3r_2 + r_3$

$\left[\begin{array}{ccc|c} 2 & 4 & 5 & -2 \\ 0 & -3 & -2 & -11 \\ 3 & 3 & 7 & 1 \end{array}\right]$

$-3(1)+3=0$
$-3(2)+3=-3$
$-3(3)+7=-2$
$-3(4)+1=-11$

21. $\begin{vmatrix} 3 & 5 \\ -2 & 7 \end{vmatrix} = (3)(7) - (-2)(5) = 21+10 = \boxed{31}$

22. $\begin{vmatrix} 2 & -5 \\ 4 & x \end{vmatrix} = 6$

$2x - 4(-5) = 6$

$2x + 20 = 6$

$2x = -14$

$\boxed{x = -7}$

23. $\begin{vmatrix} -1 & 2 & 1 \\ 2 & -2 & 3 \\ 3 & -1 & 0 \end{vmatrix}$

Expansion by minors (using 3rd row):

$3\begin{vmatrix} 2 & 1 \\ -2 & 3 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} + 0\begin{vmatrix} -1 & 2 \\ 2 & -2 \end{vmatrix}$

$= 3(6+2) + 1(-3-2) + 0(2-4)$

$= 24 - 5 = \boxed{19}$

Diagonals:

$-6 + 3 + 0 = -3$

$\begin{vmatrix} -1 & 2 & 1 & -1 & 2 \\ 2 & -2 & 3 & 2 & -2 \\ 3 & -1 & 0 & 3 & -1 \end{vmatrix} = 16 - (-3) = \boxed{19}$

$0 + 18 - 2 = 16$

24. $\left[\begin{array}{cc|c} 2 & -3 & 5 \\ 1 & 4 & -7 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_2} \left[\begin{array}{cc|c} 1 & 4 & -7 \\ 2 & -3 & 5 \end{array}\right] \xrightarrow{R_2 = -2r_1 + r_2}$

$\left[\begin{array}{cc|c} 1 & 4 & -7 \\ 0 & -11 & 19 \end{array}\right] \xrightarrow{R_2 = \frac{r_2}{-11}} \left[\begin{array}{cc|c} 1 & 4 & -7 \\ 0 & 1 & -19/11 \end{array}\right] \xrightarrow{R_1 = -4r_2 + r_1}$

$\left[\begin{array}{cc|c} 1 & 0 & -1/11 \\ 0 & 1 & -19/11 \end{array}\right]$

$\boxed{(-1/11, -19/11)}$

25. $\begin{cases} 2x-y+z=3 \\ x-y-z=4 \\ x+2y-2z=1 \end{cases}$

$$D=\begin{vmatrix} 2 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & -2 \end{vmatrix}=10$$

$$D_z=\begin{vmatrix} 2 & -1 & 3 \\ 1 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix}=2\begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix}-(-1)\begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix}+3\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$$

$$=2(-9)+1(-3)+3(3)$$

$$=-18-3+9=-12$$

$$z=\frac{D_z}{D}=\frac{-12}{10}=\boxed{\frac{-6}{5}}$$

26. $\begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix}+\begin{bmatrix} -3 & 4 \\ -2 & 7 \end{bmatrix}=\boxed{\begin{bmatrix} -1 & 5 \\ 2 & 4 \end{bmatrix}}$

27. $3\begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix}-2\begin{bmatrix} -3 & 4 \\ -2 & 7 \end{bmatrix}=\begin{bmatrix} 6 & 3 \\ 12 & -9 \end{bmatrix}-\begin{bmatrix} -6 & 8 \\ -4 & 14 \end{bmatrix}=\boxed{\begin{bmatrix} 12 & -5 \\ 16 & -23 \end{bmatrix}}$

28. $I_2-4A=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-4\begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}-\begin{bmatrix} 8 & 4 \\ 16 & -12 \end{bmatrix}=\boxed{\begin{bmatrix} -7 & -4 \\ -16 & 13 \end{bmatrix}}$

29. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \end{bmatrix}\cdot\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}=\begin{bmatrix} 1(1)+2(-1)+3(2) & 1(2)+2(0)+3(4) \\ 0(1)+(-1)(-1)+4(2) & 0(2)+(-1)(0)+4(4) \end{bmatrix}$

$$=\boxed{\begin{bmatrix} 5 & 14 \\ 9 & 16 \end{bmatrix}}$$

$2\times3 \quad 3\times2 \quad =2\times2$

30. $\begin{bmatrix} 1 & 0 \\ -4 & 5 \\ 3 & -9 \end{bmatrix}\cdot\begin{bmatrix} 2 & 3 & 5 \\ 11 & 2 & -7 \\ 8 & 0 & 1 \end{bmatrix}$

$3\times2 \quad 3\times3$

Don't match

Cannot multiply

31. $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ $\quad A^{-1} = \frac{1}{\begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix}} \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{14} \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} 2/7 & 1/14 \\ -1/7 & 3/14 \end{bmatrix}}$

or $\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array}\right] \xrightarrow{R_1 = r_1 - r_2} \left[\begin{array}{cc|cc} 1 & -5 & 1 & -1 \\ 2 & 4 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_2 = -2r_1 + r_2} \left[\begin{array}{cc|cc} 1 & -5 & 1 & -1 \\ 0 & 14 & -2 & 3 \end{array}\right] \xrightarrow{R_2 = r_2/14} \left[\begin{array}{cc|cc} 1 & -5 & 1 & -1 \\ 0 & 1 & -1/7 & 3/14 \end{array}\right]$

$\xrightarrow{R_1 = 5r_2 + r_1} \left[\begin{array}{cc|cc} 1 & 0 & 2/7 & 1/14 \\ 0 & 1 & -1/7 & 3/14 \end{array}\right]$ $\quad \boxed{A^{-1} = \begin{bmatrix} 2/7 & 1/14 \\ -1/7 & 3/14 \end{bmatrix}}$

32. $\begin{cases} x + 2y + 3z = 2 \\ x + y + z = -3 \\ -x + y + 2z = 4 \end{cases}$ $\quad A^{-1} = \begin{bmatrix} 1 & -1 & -1 \\ -3 & 5 & 2 \\ 2 & -3 & -1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$

$X = A^{-1} \cdot B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -3 & 5 & 2 \\ 2 & -3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} (1)(2)+(-1)(-3)+(-1)(4) \\ (-3)(2)+(5)(-3)+(2)(4) \\ (2)(2)+(-3)(-3)+(-1)(4) \end{bmatrix} = \begin{bmatrix} 1 \\ -13 \\ 9 \end{bmatrix}$$

$\boxed{(1, -13, 9)}$

Sorry - the partial fraction problems turned out nasty!

33. $\frac{3x-5}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$

$3x - 5 = A(x+3) + B(x-2)$

Let $x = -3$: $\quad 3(-3) - 5 = A(-3+3) + B(-3-2)$

$-14 = -5B \quad B = 14/5$

Let $x = 2$: $\quad 3(2) - 5 = A(2+3) + B(2-2)$

$1 = 5A \quad A = 1/5$

$\boxed{\frac{1/5}{x-2} + \frac{14/5}{x+3}}$

34. $\dfrac{x}{x^2+7x+12}$ $\dfrac{x}{(x+4)(x+3)} = \dfrac{A}{x+4} + \dfrac{B}{x+3}$

$x = A(x+3) + B(x+4)$

Let $x=-3$: $-3 = A(-3+3) + B(-3+4)$

$-3 = B$

Let $x=-4$: $-4 = A(-4+3) + B(-4+4)$

$-4 = -A \quad A = 4$

$$\boxed{\frac{4}{x+4} + \frac{-3}{x+3}}$$

35. $\dfrac{3x^2-2x+4}{(x+1)(x-1)^2} = \dfrac{A}{x+1} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

$3x^2-2x+4 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$

Let $x=1$: $3(1)^2-2(1)+4 = A(1-1)^2 + B(1+1)(1-1) + C(1+1)$

$5 = 2C \quad C = 5/2$

Let $x=-1$: $3(-1)^2-2(-1)+4 = A(-1-1)^2 + B(-1+1)(-1-1) + C(-1+1)$

$9 = 4A \quad A = 9/4$

Let $x=0$: $3(0)^2-2(0)+4 = A(0-1)^2 + B(0+1)(0-1) + C(0+1)$

$4 = A - B + C$

$4 = 9/4 - B + 5/2$

$B = 9/4 + 10/4 - 16/4 = 3/4$

$$\boxed{\frac{9/4}{x+1} + \frac{3/4}{x-1} + \frac{5/2}{(x-1)^2}}$$

36. $\dfrac{2x+1}{(x+3)(x^2-3x+9)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2-3x+9}$

$2x+1 = A(x^2-3x+9) + (Bx+C)(x+3)$

Let $x=-3$: $2(-3)+1 = A((-3)^2-3(-3)+9) + (B(-3)+C)(-3+3)$

$-5 = 27A \quad A = -5/27$

Let $x=0$: $2(0)+1 = A(0^2-3(0)+9) + (B(0)+C)(0+3)$

$1 = 9A + 3C$

$1 = 9(-5/27) + 3C$

$1 = -5/3 + 3C$

$8/3 = 3C \qquad C = 8/9$

Let $x = 1$: $\quad 2(1)+1 = A(1^2 - 3(1) + 9) + (B(1) + C)(1+3)$

$3 = 7A + (B+C)(4)$

$3 = 7A + 4B + 4C$

$3 = 7(-5/27) + 4B + 4(8/9)$

$3 = -35/27 + 4B + 32/9$

$4B = 81/27 + 35/27 - 96/27$

$4B = 20/27 \qquad B = 5/27$

$$\frac{-5/27}{x+3} + \frac{5/27\,x + 8/9}{x^2 - 3x + 9}$$